Complex Number 1

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$(a,b) \in \mathbb{R}^2 \Rightarrow a \in \mathbb{R},\ b \in \mathbb{R}$

↳ 2-tuple of real numbers

Identity element of '+' is 0

" " " '$*$' is 1

Identity Mapping is f

$f(x) = x$

$z_1 + z_2 = z_2 + z_1 \quad \forall z_1, z_2 \in \mathbb{C}$

$(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3) \quad \forall z_1, z_2, z_3 \in \mathbb{C}$

$z + 0 = z = 0 + z \quad \Rightarrow 0 = (0,0)$

$z + (-z) = 0 = (-z) + z \qquad \underset{(x,y)}{z}$ additive inverse $\underset{(-x,-y)}{-z}$

$z_1 \cdot z_2 = z_2 \cdot z_1$

$(z_1 \cdot z_2) z_3 = z_1 \cdot (z_2 \cdot z_3)$

$z \cdot 1 = 1 \cdot z = z$

$z \cdot z^{-1} = 1 \Rightarrow z^{-1} = \frac{1}{z}$ when $z \neq 0 \rightarrow 0 = (0,0)$

$$\boxed{(a,b) + (c,d) = (a+c, b+d)}$$

$$\boxed{(a,b)(c,d) = (ac-bd, bc+ad)}$$

$z = (x,y) \qquad (x,y)(x,y)^{-1} = 1 = (1,0)$

$\hookrightarrow (x,y)(x',y') = 1 = (1,0)$

$\Rightarrow (xx' - yy', x'y + yx') = (1,0)$

$xx' - yy' = 1$

$x'y + y'x = 0 \Rightarrow x' = \frac{-y'x}{y}$

$x\frac{(-y'x)}{y} - yy' = 1$

$y'\left(\frac{-x^2 - y^2}{y}\right) = 1 \Rightarrow y' = \frac{-y}{x^2+y^2}$

$x' = \frac{x}{x^2+y^2}$

$(x,y)^{-1} = \left(\frac{x}{x^2+y^2}, \frac{-y}{x^2+y^2}\right) \longrightarrow (x,y) \in \mathbb{C}^*$

$x^2 + y^2 \neq 0 \qquad x, y \in \mathbb{R}$

$x, y \neq 0$ //

HW

$\boxed{a^0 = 1?\quad a \in \mathbb{R}}$

$z_1 \cdot (z_2 + z_3) = z_1 z_2 + z_1 z_3$

$$z^m \cdot z^k = z^{m+n} \qquad \frac{z^m}{z^n} = z^{m-n}$$

This representation is unique $\leftarrow (a, b) \in \mathbb{C} \quad (c, d) \in \mathbb{C}$

If $a \neq c$ or $b \neq d$ then $(a, b) \neq (c, d)$

$$z_1 - z_2 = 0 = (0, 0$$

$$(a, b) - (c, d) = (a - c, b - d) = (0, 0)$$

$$a - c = 0 \qquad b - d = 0$$

$$a = c \qquad b = d$$

$$(z^m)^n = z^{mn} \qquad (z_1 \cdot z_2)^m = z_1^m z_2^m \qquad \left(\frac{z_1}{z_2}\right)^m = \frac{z_1^m}{z_2^m}$$